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DYNAMIC PLASTIC SHEAR FAILURE ANALYSIS FOR AN INFINITELY LARGE PLATE WITH A CENTRED CYLINDER UNDER IMPULSIVE LOADING

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Abstract—The transverse shear failure of a circular plate with an infinitely large outside radius, made from a rigid, perfectly plastic material, is considered. The plate has a rigid central cylindrical boss which is subjected to an initial impulsive velocity which causes a circular transverse shear hinge to develop at the interface between the boss and the plate. Two models are adopted in this paper. Rotatory inertia are disregarded in the first, the second retains the influence of rotatory inertia as well as bending moments. It is illustrated that whether a shear failure occurs or not at the interface, where the initial velocity has discontinuity, depends on the nondimensional initial kinetic energy, the material property and two ratios, one is the thickness of the plate to the radius of the boss, and the second is the density ratio of the boss to the plate. It is shown by the comparison between the two models that the influence of the rotatory inertia of the plate on the shear failure are not negligible.

NOMENCLATURE

e	nondimensional kinetic energy
G	mass of the boss
H	thickness of the plate
I_r	$mH^2/12$
m	mass per unit area of the plate
M_0	$YH^2/4$, fully plastic bending moment per unit length of the plate
M_r, M_θ	radial and circumferential bending moments per unit length of the plate
Q_0	$YH/\sqrt{3}$
Q_r	transverse shear force per unit length of the plate
R	radius of the boss
s	Z/R
s'	$s+1$
V	velocity of the plate adjacent to the boss
V_0	initial impulsive velocity of the boss
V_G	velocity of the boss
w	transverse deflection of the plate
Y	uniaxial yield stress
Z	position of plastic hinge circle measured from the interface of the plate and boss
Z'	$Z+R$
α	H/R
γ	transverse shear strain
κ_r, κ_θ	radial and circumferential curvature
μ	$G/\pi R^2$
ψ	rotation of the mid-plane due to bending
$[X]$	X_2-X_1
$(\dot{\ })$	$\partial/\partial t\,(\cdot)$
$(\ )'$	$\partial/\partial r(\cdot)$.

1. INTRODUCTION

Problems of dynamic plastic response and failure of thin plates are of considerable theoretical and technical importance. Many studies involving failure analysis of beams and plates show that shear failure is a fundamental mode of structural failure under impulsive loading. Menkes and Opat (1973) observed that transverse shear at the support was one of the three basic failure modes for impulsively loaded clamped strain-rate-insensitive aluminium alloy

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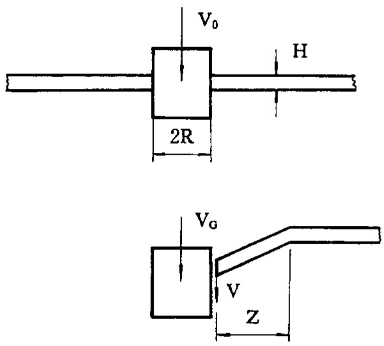

Fig. 1. Schematic illustration of the plate with a cylindrical boss.

beams. Liu and Jones (1987) conducted an experimental investigation into the dynamic plastic response and failure of strain-rate-sensitive mild-steel beams due to impact loads. They found that the beams were dominated by shear failures. Jouri and Jones (1988) have found that the transverse shear severance of beams in double shear loading occurs at a shear displacement which is much smaller than the beam thickness. Similar to Menkes and Opat's experiment, Teeling-Smith and Nurick (1991) observed three major failure modes for impulsively loaded circular plates: excessive permanent transverse deflections (mode I), tensile tearing failure (mode II) and transverse shear failure (mode III). Shen and Jones (1993) recently gave an approximate theoretical analysis for the dynamic plastic response and failure of a rigid, plastic fully clamped circular plate subjected to uniformly distributed transverse impulsive pressure. As with the circular plates, Olson *et al.* (1993) observed three similar major failure modes for blast loaded square plates.

The influence of transverse shear and rotatory inertia on the dynamic plastic response of circular plate has been theoretically examined by Jones and Gomes de Oliveira (1980). They concluded that transverse shear effects were important for small values of $(Q_0R/2M_0)$, while rotatory inertia could further decrease the maximum permanent transverse displacement up to about 14% when $(Q_0R/2M_0) > 1.5$. More recently, Yu and Zhao (1993) have examined the influence of rotatory inertia of a beam on the dynamic plastic shear failure of a cantilever with an attached mass block under impulsive loading. The shear failure occurred at the interface between the block and the beam tip because of the initial velocity discontinuity. Yu and Zhao found that the consideration of the rotatory inertia of the beam increased the kinetic energy required to cause the same shear failure.

The dynamic plastic response and failure analysis of large thin plates with a centred boss is of interest for safety design and protection in many civil and military applications. A study of the static case was made by Markowitz and Hu (1964) to consider the loading capacity of a simply supported and fully clamped orthotropic circular plate with a centred rigid boss. The current work is concerned with the dynamic plastic failure of an infinitely large thin plate with a centred boss; the plate is made of a rigid, perfectly plastic material, which is assumed to obey the simplified yield criterion shown in Fig. 3 and its associated flow rules. Two models are used in the present paper: rotatory inertia are disregarded in the first one, but considered in the second. A comparison is made between the two cases. This study can be used to assess the influence of rotatory inertia on the dynamic plastic response and failure of structures.

2. MODEL WHEN ROTATORY INERTIA ARE DISREGARDED

A thin plate of sufficiently large outer edge, as shown in Fig. 1, is centred with a rigid cylindrical boss. The initial velocity of the cylindrical boss is denoted as V_0. When the rotatory inertia are disregarded, the plate is considered as bending around a fixed plastic hinge circle at a distance Z from the interface between the plate and the boss. Sliding occurs at the interface immediately when the cylindrical boss obtains the initial velocity. Let V_G

denote the sliding velocity of the boss at $t > 0$, which is greater than the adjacent plate velocity V. The use of conservation of momentum on either side of the interface gives

$$GV_0 - GV_G = \int_R^{R+Z} 2\pi rm \frac{Z+R-r}{Z} V \mathrm{d}r = \int_0^t 2\pi R \frac{YH}{\sqrt{3}} \mathrm{d}t \tag{1}$$

which leads directly to

$$V_G = V_0 - \frac{2\pi R}{G} \frac{YH}{\sqrt{3}} t \tag{2}$$

$$V = \frac{1}{m} \frac{2R}{Z(R+Z/3)} \frac{YH}{\sqrt{3}} t. \tag{3}$$

Application of the principle of conservation of angular momentum on the system gives

$$\int_0^t 2\pi R \frac{YH}{\sqrt{3}} Z \mathrm{d}t + \int_0^t 2\pi(R+Z) \frac{H^2 Y}{4} \mathrm{d}t + \int_0^t 2\pi R \frac{H^2 Y}{4} \mathrm{d}t = \int_R^{R+Z} 2\pi rm \frac{(Z+R-r)^2}{Z} \mathrm{d}r$$

i.e.

$$VZ^2\left(R + \frac{Z}{4}\right) = \frac{3}{m}(2R+Z)\frac{H^2 Y}{4} t, \tag{4}$$

where $H^2Y/4$ and $YH/\sqrt{3}$ are the dynamic fully plastic bending moment per unit length and the dynamic fully plastic transverse shear force per unit length, respectively. Equations (3) and (4) can be combined to give a quadratic equation for the nondimensional stationary plastic hinge circle position s as

$$(2-\sqrt{3}\alpha)s^2 + (8-5\sqrt{3}\alpha)s - 6\sqrt{3}\alpha = 0, \tag{5}$$

where $s = Z/R$, $\alpha = H/R$.

The nondimensional hinge position s is then

$$s = \frac{\sqrt{(8-\sqrt{3}\alpha)^2 - 16\sqrt{3}\alpha} - (8-5\sqrt{3}\alpha)}{2(2-\sqrt{3}\alpha)} \tag{6}$$

which shows that the nondimensional position of the plastic hinge circle s depends only on the nondimensional parameter α—the ratio of plate thickness to the radius of the cylindrical boss.

The relative sliding velocity of the interface may be written in the form

$$[V] = V_0 - \left[\frac{2\pi R}{G} \frac{YH}{\sqrt{3}} + H(\alpha)\frac{Y}{m}\right] t, \tag{7}$$

where

$$H(\alpha) = \frac{2\sqrt{3}\alpha(2-\sqrt{3}\alpha)^2}{8-\sqrt{3}\alpha-3\alpha^2-(1-\sqrt{3}\alpha)\sqrt{(8-\sqrt{3}\alpha)^2-16\sqrt{3}\alpha}}.$$

Equation (7) shows that the sliding velocity at the interface decreases linearly with time from its initial value V_0.

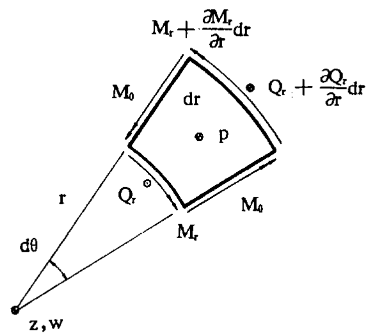

Fig. 2. Elements of the circular plate.

The total relative sliding displacement between the boss and its adjacent plate can be written as

$$[h] = K_0\left[2\pi R\frac{YH}{\sqrt{3}} + H(\alpha)\frac{G}{m}Y\right]^{-1}, \tag{8}$$

where $K_0 = GV_0^2/2$ is the initial kinetic energy of the cylindrical boss.

Failure is considered to have happened when

$$[h] \geqslant kH,$$

where k is a material constant to be determined by experiment. Complete severance occurs at the interface when $k = 1$, but transverse shear failure is likely to develop for a smaller value of k for beams (Jouri and Jones, 1988). The same situation is likely to occur for plates; there has been no experimental investigation concerning the determination of material constant k as far as we are aware. The value of k may be larger for ductile materials, and smaller for brittle materials. For convenience $k = 1$ is used here; for other values of k it can be treated in the same manner. It is evident, therefore, that complete shear failure occurs at the interface when

$$[h] \geqslant H. \tag{9}$$

The initial kinetic energy required to cause such failure is given by

$$K_0 \geqslant \left(2\pi R\frac{YH}{\sqrt{3}} + H(\alpha)\frac{G}{m}Y\right)H. \tag{10}$$

For convenience, a nondimensional kinetic energy is introduced by

$$e_0 = K_0/(HM_0) = 2GV_0^2/YH^3. \tag{11}$$

Supposing that the mass of the boss per unit horizontal section is μ, the critical value of the nondimensional kinetic energy e_{cr} can then be written as

$$e_{cr} = \frac{4\pi}{\alpha^2}\left[\frac{2}{\sqrt{3}}\alpha + H(\alpha)\frac{\mu}{m}\right]. \tag{12}$$

It is evident from eqn (12) that the nondimensional critical kinetic energy depends on two ratios. The first is the ratio of plate thickness to the radius of the cylindrical boss, and the second is the ratio of the mass of plate per unit area to that of the boss per unit horizontal section area.

3. MODEL WITH ROTATORY INERTIA

The equations of motion for the dynamic behaviour of the element of an axisymmetrically loaded circular plate, as shown in Fig. 2, can be written in the form

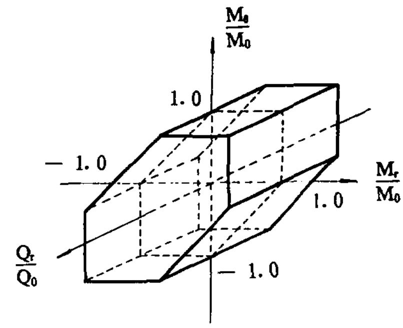

Fig. 3. Simplified yield surface.

$$(rM_r)' - M_\theta = rQ_r - I_r\ddot{\psi}r \tag{13a}$$

$$(rQ_r)' = mr\ddot{w}, \tag{13b}$$

where $I_r = mH^2/12$, m is the mass per unit area of plate, $w' = \psi + \gamma$, ψ is the rotation of lines which were originally perpendicular to the initial mid-plane due to bending and

$$\gamma = w' - \psi, \quad \kappa_r = \psi', \quad \kappa_\theta = -\frac{\psi}{r} \tag{13c}$$

are the transverse shear strain, radial curvature change and circumferential curvature change, respectively.

The discontinuity conditions for the stationary plastic hinge may be written as (Jin, 1988)

$$[M_r] = [Q_r] = [\ddot{w}] = [\dot{\psi}] = [\dot{\gamma}] = 0, \tag{14}$$

where $[X] = X_2 - X_1$.

The simplified yield criterion shown in Fig. 3, which was used by Jones and Gomes de Oliveira (1980) to examine the dynamic plastic response of circular plate with transverse shear and rotatory inertia, is adopted in this paper (see Fig. 4).

If rotatory inertia are considered the plate is divided into three different regions according to the deformation profiles. These are now considered.

1. $r \in [R, R+Z]$. There is a stationary plastic hinge circle at $r = Z+R$. The yield and geometrical conditions in this region are

$$M_\theta = M_0, \quad -Q_0 \leqslant Q_r \leqslant 0, \quad 0 \leqslant M_r \leqslant M_0 \tag{15a}$$

$$\dot{\gamma} = 0, \quad \dot{\kappa}_r = \dot{\psi}' = 0, \tag{15b}$$

respectively.

2. $r \in [R+Z, R+Z_1]$. In this plastic region the yield and geometrical conditions are

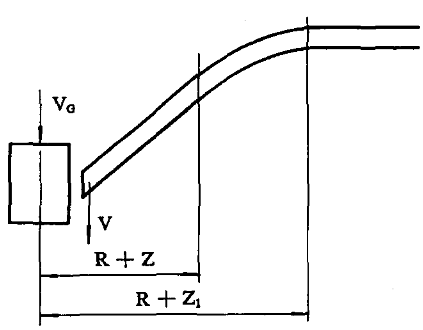

Fig. 4. Schematic illustration of the deformation profile with shear and rotatory inertia.

$$M_\theta = M_0, \quad M_r = 0, \quad -Q_0 \leqslant Q_r \leqslant 0 \tag{16a}$$

$$\dot{\gamma} = 0, \tag{16b}$$

respectively.

3. $r > R+Z_1$. This is a rigid stationary region.

3.1. $r \in [R, R+Z]$

Integrating (15b) and substituting it into (13c) yields

$$\dot{w} = C_1 r + C_2 \tag{17a}$$

$$\ddot{w} = \dot{C}_1 r + \dot{C}_2. \tag{17b}$$

Substituting (17b) into (13b) gives

$$rQ_r = -RQ_0 + m\dot{C}_1 \frac{r^3 - R^3}{3} + m\dot{C}_2 \frac{r^2 - R^2}{2}, \tag{18}$$

where the integration constant has been determined by

$$Q_r = -Q_0 \quad \text{at} \quad r = R.$$

Equations (13a) and (18) give

$$rM_r = rM_0 - rRQ_0 + R^2 Q_0 + \frac{m\dot{C}_1}{24}[2r^4 + 6R^4 - 8R^3 r - H^2(r^2 - R^2)] + \frac{m\dot{C}_2}{6}(r^3 - 3R^2 r + 2R^3), \tag{19}$$

where the integration constant has been determined by

$$M_r = M_0 \quad \text{at} \quad r = R.$$

3.2. $r \in [R+Z, R+Z_1]$

Equation (16b) predicts

$$\ddot{\psi} = \ddot{w}'. \tag{20}$$

Substituting (20) and (16a) into (13a) yields

$$r^2 \ddot{w}'' + r\ddot{w}' - \frac{12}{H^2} r^2 \ddot{w} = 0. \tag{21}$$

Since $\ddot{w} = 0$ for $Z_1 \to \infty$, the solution of (21) can be written in the form

$$\ddot{w} = \dot{C}_3 K_0\left(2\sqrt{3}\frac{r}{H}\right), \tag{22}$$

where $K_0(2\sqrt{3}(r/H))$ is the modified Bessel function of the second kind of order zero.

Integrating (22) with respect to time yields

$$\dot{w} = C_3 K_0\left(2\sqrt{3}\frac{r}{H}\right)+D(r). \tag{23a}$$

Since $\dot{w} = 0$ for $r \to \infty$ and $K_0(2\sqrt{3}(r/H)) \to 0$ as r is sufficiently large, then

$$D(r) = 0. \tag{23b}$$

Equations (23), (20) and (13a) give

$$rQ_r = -M_0 - \frac{mH}{2\sqrt{3}}\dot{C}_3 r K_1\left(2\sqrt{3}\frac{r}{H}\right), \tag{24}$$

where $K_1(2\sqrt{3}(r/H))$ is the modified Bessel function of the second kind of order one.

3.3. *Determination of the undetermined constants*

By the discontinuity conditions (14) we have

$$\dot{C}_1 Z' + \dot{C}_2 - \dot{C}_3 K_0\left(2\sqrt{3}\frac{Z'}{H}\right) = 0$$

$$C_1 + C_3\frac{2\sqrt{3}}{H}K_1\left(2\sqrt{3}\frac{Z'}{H}\right) = 0$$

$$\dot{C}_1\frac{Z'^3-R^3}{3} + \dot{C}_2\frac{Z'^2-R^2}{2} + \dot{C}_3\frac{H}{2\sqrt{3}}Z'K_1\left(2\sqrt{3}\frac{Z'}{H}\right) = \frac{RQ_0-M_0}{m}$$

$$\frac{\dot{C}_1}{12}\left(Z'^2+2Z'R+3R^2-\frac{H^2}{2}\frac{Z'+R}{Z'-R}\right)+\dot{C}_2\frac{Z'+2R}{6} = \frac{RQ_0}{m(Z'-R)} - \frac{Z'M_0}{m(Z'-R)^2},$$

where $Z' = Z+R$.

Solving the above four equations gives

$$R\dot{C}_1 = -\frac{6\alpha}{s'-1}\frac{Y}{m}\frac{f_1(\alpha,s')}{f(\alpha,s')} \tag{25a}$$

$$\dot{C}_2 = 3\frac{\alpha}{s'-1}\frac{Y}{m}\frac{f_2(\alpha,s')}{f(\alpha,s')} \tag{25b}$$

$$K_1\left(2\sqrt{3}\frac{s'}{\alpha}\right)\dot{C}_3 = \frac{\alpha}{2\sqrt{3}}\frac{6\alpha}{s'-1}\frac{Y}{m}\frac{f_1(\alpha,s')}{f(\alpha,s')}, \tag{25c}$$

where

$$f_1(\alpha,s') = \frac{2}{\sqrt{3}}(2s'+1) - \alpha\frac{s'^2+s'+1}{s'-1}$$

$$f_2(\alpha,s') = \frac{2}{\sqrt{3}}\left(3s'^2+2s'+1-\frac{\alpha^2}{2}\right)+\frac{\alpha(s'^2+1)}{4(s'-1)^2}[\alpha^2-6(s'^2-1)]$$

$$f(\alpha,s') = s'^3+3s'^2-3s'-1+\frac{\alpha^2}{2}\frac{s'^2+2s'+3}{s'-1}.$$

$s' = Z'/R$, Z' is measured from the central line of the cylindrical boss.

Omitting the lengthy details, we can obtain the nonlinear equation of s' as follows:

$$2\sqrt{3}\frac{s'}{\alpha}+\frac{K_0\left(2\sqrt{3}\frac{s'}{\alpha}\right)}{K_1\left(2\sqrt{3}\frac{s'}{\alpha}\right)}=\frac{f_3(\alpha,s')}{f_1(\alpha,s')}, \tag{26}$$

where

$$f_3(\alpha,s')=\frac{2}{\alpha}\left(3s'^2+2s'+1-\frac{\alpha^2}{2}\right)+\frac{\sqrt{3}}{4}\frac{s'^2+1}{(s'-1)^2}[\alpha^2-6(s'^2-1)].$$

Obviously, we know from eqn (26) that s', therefore s, is only dependent upon $\alpha = H/R$—the ratio of the plate thickness to the radius of the boss. Equation (26) can be solved numerically.

3.4. *Shear failure analysis*

Once the nondimensional hinge position s, as well as the integration constants are determined, the velocity of the plate adjacent to the boss can be written in the form

$$V=H_1(\alpha)\frac{Y}{m}t, \tag{27}$$

where

$$H_1(\alpha)=\frac{3\alpha}{s'-1}\frac{f_4(\alpha,s')}{f(\alpha,s')} \tag{28}$$

and

$$f_4(\alpha,s')=\frac{2}{\sqrt{3}}\left(3s'^2-2s'-1-\frac{\alpha^2}{2}\right)+\frac{\alpha(s'^2+1)}{4(s'-1)^2}[\alpha^2-6(s'^2-1)]+\frac{2\alpha(s'^2+s'+1)}{s'-1}.$$

Equation (2) still holds in this case, which means that the velocity of the boss decreases linearly with time from its initial velocity V_0. The relative velocity between the boss and its adjacent plate is given by

$$[V]=V_0-\frac{1}{G}\left(2\pi R\frac{YH}{\sqrt{3}}+H_1(\alpha)\frac{GY}{m}\right)t. \tag{29}$$

The total relative sliding displacement is therefore

$$[h]=K_0\left(2\pi R\frac{YH}{\sqrt{3}}+H_1(\alpha)\frac{GY}{m}\right)^{-1}, \tag{30}$$

where $K_0=\frac{1}{2}GV_0^2$ is the initial kinetic energy of the boss.

In the same manner as in Section 2, complete shear failure is considered to occur when $k = 1$, i.e.

$$[h]\geqslant H \tag{31}$$

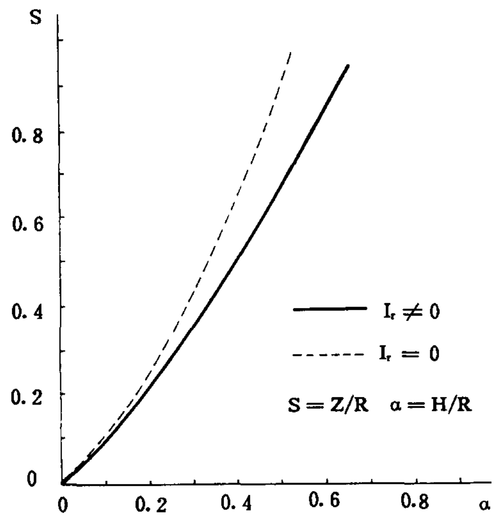

Fig. 5. s–α Curves.

i.e.

$$K_0 \geqslant H\left(2\pi R\frac{YH}{\sqrt{3}}+H_1(\alpha)\frac{GY}{m}\right). \tag{32}$$

In this case the nondimensional critical kinetic energy may be written in the form

$$e_{cr} = \frac{4\pi}{\alpha^2}\left[\frac{2}{\sqrt{3}}\alpha+H_1(\alpha)\frac{\mu}{m}\right], \tag{33}$$

where μ is the mass of the cylindrical boss per unit horizontal section area.

In a manner similar to Section 2, eqn (33) indicates that e_{cr} depends on α and the ratio of the mass of the boss per unit horizontal section area to that of the plate per unit area of mid-plane.

4. DISCUSSION

Equations (12) and (33) mean that the nondimensional kinetic energy required for causing complete shear failure at the interface depends on two ratios. The first one is the ratio of the thickness of the plate to the radius of the boss, and second is the ratio of the mass of the boss per unit horizontal section area to that of the plate per unit area of mid-plane. In order to assess the influence of rotatory inertia, we make a comparison between the two cases in Figs 5 and 6. Figure 5 shows that the nondimensional hinge position s, in the case when rotatory inertia is considered, is smaller than that when rotatory inertia are disregarded under the same ratio α (the thickness of the plate to the radius of the cylindrical boss). In addition, Fig. 6 indicates that the nondimensional critical energy in the case when rotatory inertia are considered is larger than that in the case when rotatory inertia are disregarded under same α and same μ/m. Figure 6 also shows that the difference in e_{cr} between the two cases increases with the increase of μ/m under the same α, and the difference increases with the increase of α under the same μ/m.

The influence of α on the nondimensional hinge position can be clearly seen in Fig. 5, s increases monotonically with the increase of the ratio of the plate thickness to the radius of the boss, the nondimensional kinetic energy e_{cr} decreases monotonically with the increase of the same ratio. Nevertheless, if α is fixed at a particular value, increase of the value of μ/m causes a concomitant increase of e_{cr}.

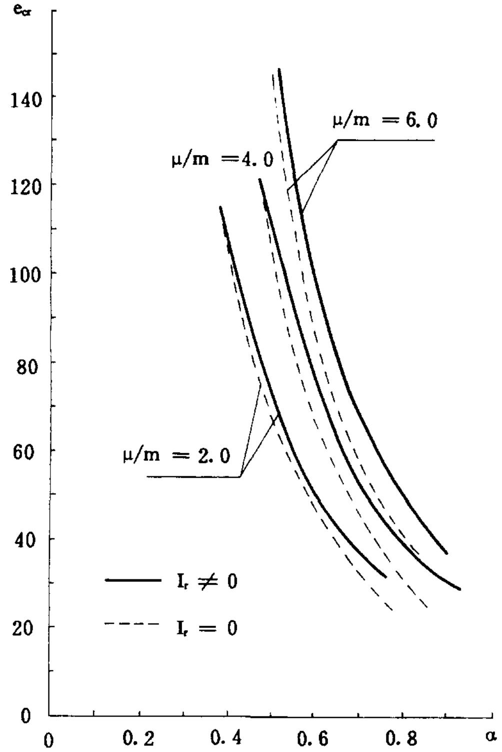

Fig. 6. The relationship between e_{cr} and α with different μ/m ($k = 1$).

5. CONCLUSION

A theoretical solution has been presented for predicting the dynamic plastic response and shear failure of a rigid, perfectly plastic infinitely large plate with a central cylindrical boss under an initial impulsive velocity. The plate is assumed to obey the simplified criterion and its associated flow rules. A comparison has been made to assess the importance of the influence of rotatory inertia included in the governing equation on the dynamic plastic shear failure of the structure. Two analytical models are used in the paper, the first is without the influence of rotatory inertia, the second retains the influence of the rotatory inertia of the plate as well as bending moments. It is shown that whether shear failure occurs or not at the interface, where the initial velocity has discontinuity, depends on the material constant k and two ratios. The first is the ratio of plate thickness to the radius of the cylindrical boss, the second is the density ratio of the boss to the plate. Consideration of the influence of rotatory inertia of the plate increases the initial kinetic energy of the boss required to cause dynamic plastic shear failure. It is interesting to note that this property coincides with that of the shear failure analysis of a cantilever with an attached mass block at its tip, the consideration of the rotatory inertia of the beam increases the kinetic energy required to cause the complete shear failure at the interface between the beam and the mass block (Yu and Zhao, 1993).

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